ENRE 620 – Mathematical Techniques of Reliability Engineering
Summer II 2019

Instructor: Dr. Reuel Smith

Office Hours: Wednesday 3:00PM – 5:00PM or by appointment

Course Description: Mathematical Techniques in Reliability Engineering is an introduction to reliability engineering and several mathematical techniques, principles, and programming applications used in the field. ***Basic knowledge of calculus is recommended and required for this course*** as much of the material will be covering techniques that go beyond that knowledge. The course will train students in recognizing and setting up basic reliability engineering problems using the mathematical techniques covered in preparation for advanced courses in the reliability engineering program. These techniques include (but are not limited to) maximum likelihood estimation and linear regression analysis, ordinary linear differential equation analysis, Laplace Transforms, and numerical integration. The course will also introduce application of these mathematical techniques within the scope of common software used in modern reliability analysis including Excel, MATLAB and OpenBUGs which is supplemented by in-class tutorials, discussion, and reading.

Course Structure: The six-week long course will be of a seminar style and consist mainly of lectures, discussions, examples and applications, and case studies. The classes are held Monday and Wednesday from 5:15PM to 9:00PM. Online students are encouraged to watch lectures live in order to participate in the in-class discussions, exercises, and activities. The overall structure of the course consists of seven main topics and several subtopics.

Module 1: Introduction to Probability and Statistics

This is an introduction to certain topics in probability and statistics that are used in reliability engineering. The basics of probability are covered as well as the concept of discrete and continuous distributions. The basics of statistics are also covered including statistical elements, maximum likelihood estimation, uncertainty analysis, distribution fitting tests, regression analysis, and distribution plotting.

Module 2: Introduction to Advanced Reliability Engineering Techniques

The module will go over some review of Eigen Analysis which covers eigenvectors and eigenvalues as well as application in reliability engineering. The module also covers multi chain Monte Carlo (MCMC) assisted Bayesian analysis which involves understanding of different models, coding, and application through useful engineering tools.

Module 3: Introduction to Ordinary Differential Equations

This is the introduction to ordinary differential equations (ODEs) as well as a brief overview on how they are applied to reliability engineering. ODE's of several orders are covered including first, second, and multiple. With that the class will learn different forms of ODEs, how to set up a differential equation based on the physical properties of an engineering problem, and how to solve the equation using manual calculations and computational techniques aided by MATLAB routines.

Module 4: Power Series Solutions to Ordinary Differential Equations

Differential equations that have variable coefficients cannot be solved using standard algebra. For this there are series solutions that use methods such as the Power Series to form coefficients needed to solve the differential equation. This module goes over some of the more common series solutions as well as more advanced forms of differential equation formats.

Module 5: Laplace Transforms and Fourier Analysis

In addition to algebraic solutions, differential equations and the engineering problems thereof can be solved using Laplace Transforms, operations that involves the simplification of complex differential equations and that is popular among engineers. This module will list several known Laplace Transforms and go over the methods used to solve differential equations using these transforms. The Fourier Series is also discussed in how they are used to solve complex differential equations. Several examples will be addressed that uses direct the Fourier Series as a solution method as well as Fourier Transforms that operate similarly to Laplace Transforms.

Module 6: Partial Differential Equations

This module introduces partial differential equations (PDEs) which are typically used to model more complex physical systems than ODEs which are reserved for simple physical systems. A set of simple physical systems and their relation to PDEs will be introduced and several case studies will be performed.

Module 7: Numerical Methods and Integration Techniques

The final module covers numerical methods that may be used to solve differential equations and perform integration. The module will cover several numerical integration algorithms as well as several MATLAB scripts to aid in application.

Tentative Timeline

Because of the interactive structure of the course, it is strongly recommended that the following chapters and sections be read prior to the start of the course.

	Date	Class Topics	Reading
Day 1	July 8th	Module 1: Introduction to Probability and Probability Models and Identification Activity	Chapter 2 from Modarres, Kaminskiy, and Krivstov
Day 2	July 10th	Module 1: Bayesian Theorem Applications and Statistical and Distribution Fitness Analysis	Chapter 2, 3.1.1, 3.3.1, & 3.3.2 from Modarres, Kaminskiy, and Krivstov
Day 3	July 15th	Quiz 1 (Module 1) Module 2: Eigen Analysis and MCMC	Chapter 4(intro), 4.1, 7, & 8 (skip 8.5) from Kreyszig
Day 4	July 17th	Module 3: Elementary Techniques for Ordinary Differential Equations and Introduction to Linear Differential Equations (PART 1)	Chapter 1.1, 1.3, 1.4, 1.5, 2.1, 2.2, 2.5, & 2.7 from Kreyszig
Day 5	July 22nd	Quiz 2 (Module 2) Module 3: Introduction to Linear Differential Equations (PART 2) and Systems of Linear Differential Equations	Chapter 3 from Kreyszig
Day 6	July 24th	Module 4: Power Series Fundamentals and Legendre Equations and other Power Series Based Equations and Methods	Chapter 5 (5.5 optional) from Kreyszig
Day 7	July 29th	Quiz 3 (Module 3 & 4) Module 5: Laplace Transform Fundamentals and Applications Mid-semester evaluation	Chapter 6 from Kreyszig
Day 8	July 31st	Module 5: Fourier Analysis Fundamentals and Applications	Chapter 11 from Kreyszig
Day 9	August 5th	Quiz 4 (Module 5) Module 6: Partial Differential Equations	Chapter 12 from Kreyszig
Day 10	August 7th	Module 6: Advanced Partial Differential Equation Applications	Chapter 12 from Kreyszig
Day 11	August 12th	Module 7: Numerical Analysis Review for Final Exam	Chapter 19.1, 19.2, 19.5, 21.1, 21.2, & 21.3 from Kreyszig
Day 12	August 14th	Final Exam (Module 6 & 7/Cumulative)	

List of Skills in ENRE 620

Module 1	A1	Simplify a Boolean expression
	A2	Identify a best estimate distribution based on the content of the problem statement
	A3	Perform a probability problem using the relationships between the PDF, CDF, and reliability function
	A4	Form a likelihood and perform a conversion between a likelihood and a log-likelihood in a closed form expression
	A5	Set up a Bayes Theorem inference based on content of a problem statement and data
	A6	Use maximum likelihood estimation to find point estimates based on data
	A7	Use regression analysis and probability plotting to determine the best fit for a set of data
	A8	Generate a plotting scale that adheres to a given distribution, that is form the x and y axes for a probability plot
Module 2	B9	Identify uses for Eigen analysis in engineering
	B10	Perform Eigen analysis on a matrix of any size
	B11	Perform an Eigen analysis based on the content of a problem statement (be able to validate by MATLAB)
	B12	Identify various multi chain Monte Carlo (MCMC) techniques and how they are used in engineering
	B13	Perform Metropolis-Hastings MCMC using MATLAB
	B14	Perform Gibbs MCMC using OpenBUGs
	B15	Compare methodologies in performing MCMC
Module 3	C16	Use ordinary differential equations (ODEs) to solve an engineering based problem by way of the modeling process
	C17	Solve an ODE using separation of variables and exact reduction method. Describe the difference between the two methods.
	C18	Solve a homogeneous and nonhomogeneous linear differential equation (LDE) by hand.
	C19	Solve a system of LDEs two ways: (1) one-by-one and (2) as a group
	C20	Set up a Markov Analysis by identifying Markov states and drawing a Markov diagram
	C21	Use Markov Analysis and elementary LDEs to solve a systems reliability problem
Module 4	D22	Solve an ODE or LDE using power series
	D23	Express and plot a probability distribution as a power series
	D24	Code and perform a for-loop operation in MATLAB and successfully solve an associated problem
	D25	Plot the Legendre polynomial of a degree n
	D26	Solve a set of ODEs or LDEs using Frobenius method
Module 5	E27	Find the Laplace transform of a function $f(t)$ and similarly the inverse Laplace transform of a function $F(s)$.
	E28	Identify common uses for Laplace transforms in engineering.
	E29	Use Laplace transforms to solve an ODE or LDE.
	E30	Solve an engineering based problem using Laplace transforms
	E31	Find the Fourier transform
	E32	Perform a short literature review to find practical uses of the Fourier transform in reliability or other types of engineering.
Module 6	F33	Identify the three main partial differential equation (PDE) types and state some of their uses
	F34	Derive by hand the expression for displacement from any PDE
	F35	Plot the modes of a given displacement from any PDE
	F36	Animate the motion of a given displacement from any PDE using MATLAB or another program
Module 7	G37	Test and modify numerical method algorithms for a given situation
	G38	Identify the effectiveness of a numerical method using additive or multiplicative error
	G39	Develop a function from scratch in MATLAB to execute a numerical method
	G40	Execute a reliability analysis using numerical integration methods

Grading: The grading of the course will be primarily based on achievements/skills obtained during a series of quizzes throughout the semester and a final exam (August 17th). There will be homework assigned on a weekly basis, but these will be for practice and training and thus will be ungraded. Homework is made up of some training problems that can be done either to be done by hand or coded in MATLAB. I recommend working homework by hand initially and confirming any work by MATLAB. I also recommend setting up study groups amongst yourselves. Try setting up meeting times and places for after class as early as possible. **Please note that the grade is *not* as important as your overall understanding of particular skills in the course and your overall well-being**.

- Quizzes
  - Dates to be determined (about five)
  - Open book, open note, closed internet
  - Test period approximately 30 minutes per quiz
- Final Exam
  - Date: August 17th
  - Open book, open notes, closed internet
  - Test period approximately 2-3 hours

Required Textbooks:

- Reliability Engineering and Risk Analysis: A Practical Guide (3rd Ed.) by Mohammad Modarres, Mark P. Kaminskiy, and Vasiliy Krivtsov

- Advanced Engineering Mathematics, Wiley, 2011. (10th Ed.) by Erwin Kreyszig

Recommended Textbooks:

- Advanced Engineering Mathematics with Mathematic and MATLAB, 1998. (Vol. 1 & 2) by Reza Malek-Madani

- Probability Distributions Used in Reliability Engineering, 2016 by Andrew O'Conner, Mohammad Modarres, and Ali Mosleh

Recommended Tools and Applications

- A laptop and Skype for site-to-site discussion
- MATLAB (download on Terpware)
- OpenBUGs
- R Software